Evaluation activities of JCPRG: Analysis of $^6,^7$Li+n reaction

Hokkaido University
D. Ichinkhorloo, N. Furutachi, M. Odsuren,
M. Aikawa and K. Kato,

Collaborators:
T. Matsumoto (Kyushu Univ.),
Y. Hirabayashi (Hokkaido Univ.),
S. Chiba (Tokyo Inst. of Tech.),
K. Yamamoto (Niigata Univ.),
M. Ohta (Konan Univ.),
H. Masui (Kitami Inst. Tech.)

The 3rd Asian Nuclear Reaction Database Development Workshop, 27-29 August, 2012
• **Introduction:**
  Evaluation activities of JCPRG

Evaluation of nuclear reaction data for
- Nuclear fusion engineering
  - $^{6,7}$Li($n,n'$) reaction data
- Nuclear medical science
  - $^9$Be($p,n)^9$B reaction data
- Nucleosynthesis
  - $^{17}$O($n,\gamma)^{18}$O reaction data
- Investigation of nuclear structure
  - Reaction cross section of Neutron-rich Carbon isotopes
Analysis of resonance state of $^9$Be and $^9$B

- **Boron Neutron Capture Therapy**
  $$^{10}\text{B}(n,\gamma\alpha)^7\text{Li}$$
  - As a neutron source of this reaction, Evaluation of $^9\text{Be}(p,n)^9\text{B}$ reaction is necessary.
  - For this purpose, it is needed to establish reliable method to describe the structures of $^9\text{Be}$ and $^9\text{B}$.
  - Structures of $^9\text{Be}$ and $^9\text{B}$ have been calculated.

- **Method: 3-body ($2\alpha + p(n)$)**
  - OCM+CSM
  - To discuss $1/2^+$ state of $^9\text{Be}$, methods of ACCC and CLD were adopted.
Evaluation of $^{17}$O+n reaction

- Motivation
  In the s-process nucleosynthesis, the $^{17}$O(n,γ)$^{18}$O reaction becomes the Bypass to create the elements heavier than Oxygen isotopes. Since there is no measurement for the cross section of the $^{17}$O(n,γ) reaction is necessary.

- Model
  - Cluster orbital shell model (COSM) +CSM
  - E1 transition matrix element

\[
\langle \Phi(^{18}\text{O}) | \mathcal{O}(E1) | \Psi \rangle = \langle \Phi(^{18}\text{O}) | \mathcal{O}(E1) | \psi_0 \rangle + \sum_\alpha \left( \frac{1}{E - E_\alpha} \right) \langle \Phi(^{18}\text{O}) | \mathcal{O}(E1) U^{-1} | \chi_\alpha \rangle \left( \chi_\alpha | UV \right) \psi_0
\]

\[
\psi_0 = \left[ \Phi(^{17}\text{O}) \otimes \varphi_n \right]_J
\]
• $^{22}$C: Extremely large reaction cross section has been measured for $^{22}$C
  → Two neutron halo structure of $^{22}$C was proposed.

• To explain this reaction cross section and understand the structure of $^{22}$C, theoretical investigation is necessary.

• It is planned to calculate the reaction cross section using AMD wave function.

Calculation of reaction cross sections of neutron-rich Carbon isotopes

Y.Tanaka et al., PRL104, 062701 (2010)
Summary:

- **Nuclear medical science**
  - For the evaluation of $^9$Be(p,n)$^9$B reaction, resonance states of $^9$Be and $^9$B was analyzed in detail using 3-body cluster model with OCM and CSM.

- **Nucleosynthesis**
  - $^{17}$O(n,$\gamma$)$^{18}$O neutron capture reaction cross section was calculated using COSM+CSM.
    - COSM is applicable to other (n,$\gamma$) reaction.

- **Investigation of nuclear structure**
  - For the calculation of reaction cross sections of Carbon isotopes, structures of Carbon isotopes was calculated AMD.
Analysis of $n+^{6,7}\text{Li}$ reactions

Dagvadorj Ichinkhorloo
Meme Media Laboratory, Hokkaido University

K. Katō and M. Aikawa
Nuclear Reaction Data Centre, Faculty of Science,

Y. Hirabayashi,
Information Initiative Center, Hokkaido University,

T. Matsumoto
Department of Physics, Kyushu University

S. Chiba
Research Laboratory for Nuclear Reactors, Tokyo Institute of Technology,
The breakup of $^7$Li nucleus into $\alpha + t$ and their inverse reactions in the low relative energy region are a great interest from the astrophysics point of view.

The figure taken in to account from http://cococubed.asu.edu
Introduction

Tritium breeding blankets

- Protect the magnets and the vacuum vessel from neutron and gamma radiation.
- Produce the tritium necessary for continued fusion reactions.
- Convert neutron energy into heat and evacuate it to generate a cycle capable of supplying electricity.

\[
\begin{align*}
D + T & \rightarrow ^4\text{He} + n + \text{energy} \\
\text{Li} + n & \rightarrow ^4\text{He} + T + \text{energy} \\
D + \text{Li} & \rightarrow 2 \; ^4\text{He} + \text{energy}
\end{align*}
\]
• Introduction

- The double-differential neutron emission cross sections and the energy-angular distributions of secondary neutrons are very useful, because they include almost all the information that determines the neutron spectra in an assembly.

- The data for Li\((n, n')\) reaction are particularly important because the statistical model, which is used often in evaluation of nuclear data for medium to heavy nuclei, cannot be applied to the few-body breakup process of light nuclei.

- The theoretical model for the calculations of emission spectra from light nuclei such as lithium is not enough established at present.
• Purpose of this study

• The cross sections for $^{6,7}\text{Li}(n,n')$ $^{6,7}\text{Li} \rightarrow \alpha + d$ or $t$ reactions are evaluated by using the CDCC, in which we adopt wave functions of $^{6,7}\text{Li}$ with the $\alpha + d$ or $t$ model.

• We calculate cross section for the $n + ^{6,7}\text{Li}$ scattering with the complex JLM effective nucleon-nucleon interactions.

• The calculated elastic and inelastic cross sections and continuum neutron spectra show compared with the experimental data at 11.0 – 18.0 MeV incident neutron energy regions.
Experiments of $n + ^{6,7}\text{Li}$ reactions

- The calculations involve many parameters and then are lacking in predictability.

- Experimental data leading to $^{6,7}\text{Li}$ continuum breakup processes are extremely rare for the neutron energy region above 20 MeV. The data are needed up to 150 MeV.

•CDCC frameworks

The Continuum-Discretized Coupled-Channels method:

- Developed by Kyushu group about 30 years ago.
- As one of the most reliable methods for treating breakup processes.

The CDCC was used by many authors for investigations of $^6,^7\text{Li}$ elastic and inelastic scattering which has shown the significant effects of projectile breakup processes ($^6,^7\text{Li} \rightarrow \alpha + d$ or $t$).

Though the CDCC method has been also applied to the sequential breakup via the resonance state of $^6,^7\text{Li}$,

The CDCC method has not yet confirmed its applicability to the breakup into $\alpha$- $d$ or $t$ continuum states, especially to the breakup spectra of $^6,^7\text{Li} + n$.

The binding energy of the $1^+$ ground state is observed as 1.47 MeV with respect to the $^6\text{Li} \rightarrow \alpha + d$ threshold, and low-energy part of the $\alpha$-$d$ scattering phase shifts in the $S$-wave ($\ell =0$), $P$-wave ($\ell =1$) and $D$-wave ($\ell =2$) have been obtained experimentally by $\alpha + d$ model.
• $\alpha + d$ model in $^6\text{Li}$

$\alpha - d$ scattering phase shifts for $\ell = 0, 1$ and $2$

Yukinori Sakuragi et al,
\( \cdot \alpha + t \) model in \(^7\text{Li}\)

\[ V_{\alpha t}(r) = V_{\text{central}} + V_{\text{spin-orbit}} \]

For \(^7\text{Li}\) the S-wave (\(\ell = 0\)), P-wave (\(\ell =1\)), D-wave (\(\ell = 2\)), and F-wave (\(\ell =3\)) continua then split into \(1/2^+\), \(3/2^-\), \(1/2^-\), \(3/2^+\), \(5/2^+\) and \(7/2^-\), \(5/2^-\), respectively, for the \(\alpha-t\) scattering phase shifts.
\( \alpha + t \) model in \(^7\text{Li}\)


Exp.data: R. J. Spiger, T. A. Tombrello,Phys. Rev. 163(1967), 964
The Method of CDCC in $^6,^7\text{Li}$

The three body system is described by:

\[
H = H_{d(t)\alpha} + K_R + U,
\]

\[
U = U_{d(t)n}(r_{d(t)n}) + U_{\alpha n}(r_{\alpha n}).
\]

\[
H_{d(t)\alpha} = K_R + U_{\alpha d(t)}(r_{\alpha(t)}),
\]

\[
[K_R + K_R + V_{d(t)\alpha}(r) + U_{nd(t)}(r_{d(t)n}) + U_{n\alpha}(r_{n\alpha}) - E]\Psi_{JM}(\vec{r}, \vec{R}) = 0
\]
The Method of CDCC in $^6$Li

After the discretization, the CDCC wave function is given by

$$\Psi_{JM}^{CDCC}(\vec{r}, \vec{R}) = \sum_{L} Y_{JM}^{\ell_0l_0L} \phi_0(r) \chi_{\ell_0l_0L}^{\gamma_0}(P_0, R)/R + \sum_{\ell, I, L} \sum_{i=1}^{N} \sum_{\ell=0}^{\ell_{max}} \sum_{I} Y_{JM}^{\ell IL} \phi_{i\ell I}(r) \chi_{\ell IL}^{\gamma}(P, R)/R,$$
The model space of $^6\text{Li}$

$\ell = 0$

$\ell = 2$

$^1S_1$

$^1D_1$

$^2D_1$

$^3D_1$

Excitation energy of $^6\text{Li}$ [MeV]

-1.47 MeV
• The model space of $^7$Li

Excitation energy of $^7$Li, [MeV]

-2.475  
-1.986

$\ell = 1$

$^3P_1$

$^1P_1$

$^1S_1$

$\ell = 2$

$^3D_1$

$^5D_1$

$\ell = 3$

$^5F_1$

$^7F_1$

$\alpha$

$r, \ell$

$t$

$s_z = 1/2$

$^{7}$Li

$^{7}$Li + 2n

12.64

$^{5}$Li + 2n

10.24

$^{3} \frac{5}{2}, \frac{3}{2}$

9.570

$^{5}$He + d

8.75

$^{3} \frac{3}{2}, \frac{1}{2}$

7.454

$^{6}$Li + n

6.604

$^{5} \frac{5}{2}, \frac{1}{2}$

4.652

$^{4}$He + t

0.47761

$^{1} \frac{1}{2}, \frac{1}{2}$

$J^P = \frac{3}{2}, T = \frac{1}{2}$
\[ n \rightarrow 6,7\text{Li-}n \text{ potential} \]

We use the complex Jeukenne-Lejeune-Mahaux effective nucleon-nucleon (JLM) interaction based on a single-folding model.

\[ \nu_{j_0}(E, \rho, R_{j_0}) = \lambda_v V(\rho, E) \exp(-R_{j_0}^2 / t_R^2) + i\lambda_w W(\rho, E) \exp(-R_{j_0}^2 / t_I^2), \]

\[ t_R = t_I = 1.2, \lambda_v = 1.0 \text{ and } \lambda_w = 0.1 \rightarrow \lambda_w \text{ is optimized} \]

**Coupling potential:**

\[ V_{\gamma\gamma'}(R) = \int \rho_{\gamma\gamma'}(s, \Phi_R) \nu_{j_0}(E, \rho, R_{j_0}) ds d\Omega_R, \]

**Transition density of \(6,7\text{Li}:**

\[ \rho_{\gamma\gamma'}(s, \Phi_R) = \left\langle Y_{JM}^{\ell I} \hat{\phi}_{i'\ell'}^I \right\rangle \sum_{j \in 6or7\text{Li}} \delta(s - s_j) \left| Y_{JM}^{\ell' I'} \hat{\phi}_{i'\ell'}^I \right\rangle \]

Elastic scattering angular distributions of \( n + ^6\text{Li} \).

For potential norm. factors;
JLM: Imaginary - 0.1

Inelastic scattering angular distributions of $^6\text{Li}(n,n')^6\text{Li}^*$ ($3^+$, 2.186 MeV)

\( ^6\text{Li}(n,n') \): Neutron energy spectra for projectile energy and emission angle

The disagreement at the low energy region is due to the fact that experimental data contain contribution from the \((n,2n)\) reaction, which corresponds to 4-body break-up reaction \(^6\text{Li}(n,nnp)\alpha\).

\[ E_n \text{ [MeV]} \quad K. \text{ Kato, and S. Chiba, Phys. Rev. C 83, 064611 (2011)} \]

$^6\text{Li}(n,n')$: Neutron energy spectra for projectile energy and emission angle
Elastic scattering angular distributions of $n + ^7\text{Li}$

- **Red**: Single channel calc. (without BU effects)
- **Black**: Full channel ($S$, $P$, $D$ and $F$ waves)
- **Green**: included $P$ and $F$ waves
- **Blue**: Ground and resonance states

The optimized $\lambda_w$ is 0.1.

Breakup effect is significant

For potential norm. factors:

- **JLM**: Imaginary - 0.1
Inelastic scattering angular distributions of $^7\text{Li}(n,n')^7\text{Li}^*$ ($7/2^-$, 4.63 MeV)

Black: Full channel ($S$, $P$, $D$ and $F$ waves)
Green: included $P$ and $F$ waves
Blue: Ground and resonance states

Ibaraki et al.
Chiba et al.
Hogue et al.
\( ^{7}\text{Li}(n,n') \): Neutron energy spectra for projectile energy and emission angle

• **Summary**

  - We proposed a calculations model consisting of CDCC for $^6,^7$Li$(n, n')$ reaction.
  - We analyze $^6,^7$Li$(n,n')$ reaction by using CDCC method with JLM interaction.
  - The CDCC calculations reproduce the elastic and inelastic angular distributions comparison with experimental data.
  - It is found that the elastic cross sections for incident energies between 7.47 - 18.0 MeV can be reproduced by the present analysis with one normalization parameter for the imaginary part of the JLM interaction ($\lambda_w = 0.1$), and breakup effects on the elastic cross section are significant.
  - We found that CDCC method can apply to breakup (into $\alpha$-$d$ or $t$) neutron spectra of $^6,^7$Li$(n,n')$. 
• **Future work**

• We will calculate 4-body breakup reaction of $^6\text{Li}(n,n')\alpha n p$. It will be treatment of our present calculation at the higher

Inclusion of the 4-body process is a big theoretical challenge, but an attempt in that direction is underway.
• **Future work**

• The $^6\text{Li}+n$ model for $^7\text{Li}$ is still open problem in the $n + ^7\text{Li}$ reaction for analyze higher excitation energy of $^7\text{Li}$.

• We also interest to investigate higher neutron incident energy region with present calculation compared with the data libraries.
Thank you for your attentions
The Scattering Cross Section

Scattering Cross Sections

Total

\[ \sigma_{\text{tot}} = \frac{\text{Number of scattered neutrons per sec}}{\text{Incident neutron flux}} = \left[ \frac{\text{time}^{-1}}{\text{area}^{-1}} = \text{area} \right] \]

Differential

\[ \frac{d\sigma}{d\Omega} = \frac{\text{Number of scattered neutrons per sec into angle element } d\Omega}{\text{Incident neutron flux} \cdot d\Omega} \]

Double Differential

\[ \frac{d\sigma}{d\Omega dE'} = \frac{\text{Number of ... and with energies between } E' \text{ and } E'+dE'}{\text{Incident neutron flux} \cdot dE' d\Omega} \]
The pseudostate method

In the pseudostate method, we diagonalize in a space spanned by a finite number of $L^2$–type basis functions, say, and obtain discrete eigenstates as:

$$\hat{\phi}_{i\ell I}(r) = \sum_j A_{i\ell I,j} \varphi_{j\ell}(r)$$

Wave functions of $^{6,7}$Li:

As the basis functions, complex-range Gaussian basis functions are adopted by T.Matsumoto et al.
\[
\hat{\chi}_{\gamma_0}(P_0, R) = \chi_{\gamma_0}(P_0 / R), \quad \gamma_0 = (0, \ell_0, I_0, L, J)
\]
\[
\hat{\chi}_\gamma(\hat{P}_\gamma, R) = W_\gamma \chi_\gamma(\hat{P}_\gamma / R), \quad \gamma = (i, \ell, I, L, J).
\]

\[
\left[ \frac{d^2}{dR^2} + \hat{P}_\gamma^2 - \frac{L(L+1)}{R^2} - \frac{2\mu_n^6_{Li}}{\hbar^2} V_{\gamma \gamma}(R) \right] \hat{\chi}_\gamma(\hat{P}_\gamma, R) = \sum_{\gamma' \neq \gamma} \frac{2\mu_n^6_{Li}}{\hbar^2} V_{\gamma \gamma'}(R) \hat{\chi}_{\gamma'}(\hat{P}_{\gamma'}, R)
\]

Coupling potential:
\[
V_{\gamma \gamma'}(R) = \left\langle Y_{JM}^{\ell IL} \hat{\phi}_{i \ell I}(r) U|Y_{JM}^{\ell' I'L'} \hat{\phi}_{i' \ell' I'}(r)\right\rangle_{\bar{r}, \Omega_R}.
\]
The complex-range Gaussian basis functions are oscillating with $r$. They are therefore expected to simulate the oscillating pattern of the continuous breakup state wave functions better than the real-range Gaussian basis functions do. Moreover, numerical calculation with the complex-range Gaussians can be done using essentially the same computer programs as for the real-range Gaussians.

\[
\phi_{nl}^C(r) = r^l e^{-\eta_n r^2} \cos[b \nu_n r^2] = r^l \frac{e^{-\eta_n r^2} + e^{-\eta_n^* r^2}}{2} \\
\phi_{nl}^S(r) = r^l e^{-\eta_n r^2} \sin[b \nu_n r^2] = r^l \frac{e^{-\eta_n r^2} - e^{-\eta_n^* r^2}}{2i} \\
\eta_n = (1 + i b) \nu_n, \quad b = \frac{\pi}{2}
\]