Description of even-even triaxial nuclei in the framework of Asymmetric Rotor Model and Interacting Boson Approximation.

Vidya Devi

Meme Media Laboratory, Hokkaido University

AASPP2012@Pohang
Outline

1. Introduction survey of data-what nuclei do
2. Review on ARM and IBA
3. Symmetries in nuclei
4. Result and discussion
5. Summary
Simple observables - even-even nuclei

\[ R_{4/2} = \frac{E(4^+_1)}{E(2^+_1)} \]

\[ B(E2; 4^+_1 \rightarrow 2^+_1) \]

\[ B(E2; 2^+_1 \rightarrow 0^+_1) \]

Masses

\[ B(E2; J_i \rightarrow J_f) \equiv \frac{1}{2J_i + 1} \left| \langle \Psi_i | E2 | \Psi_f \rangle \right|^2 \]
Broad perspective on structure structural evolution: $R_{4/2}$
Sudden changes in $R_{4/2}$ signify changes in structure, usually from spherical to deformed structure.
Davydov and Filippov investigated the asymmetric rotor model. The value of asymmetry parameter ($\gamma$) can be obtained by the expression

$$\gamma = \frac{1}{3} \sin^{-1} \left[ \frac{9}{8} \left( 1 - \left( \frac{R_\gamma - 1}{R_\gamma + 1} \right)^2 \right) \right]^{1/2}$$

(1)

where

$$R_\gamma = \frac{E(2^+)}{E(2^+)}.$$ 

This asymmetric parameter is also calculated from B(E2) values

$$B(E2: 2^+_\sigma \rightarrow 0^+_1) = \frac{1}{2} \left( \frac{e^2 Q_0^2}{16\pi} \right) \left( 1 + (-1)^\sigma \frac{3 - 2 \sin^2(3\gamma)}{\sqrt{9 - 8 \sin^2(3\gamma)}} \right)$$

(2)
where \( Q_0 = 3 Z R^2 \beta / \sqrt{5\pi} \) and the value of \( \beta \) is easily calculated from eq.(2). Therefore we calculate the \( \beta \) by approximate empirical Grodzins relation

\[
E_{21^+} B(E2 : 2_1^+ \rightarrow 0_1^+) = 2.5 \times 10^{-3} Z^2 A^{-1} [MeV e^2 b^2]
\] (3)

On relating \( \beta \) and \( E_{21^+} \) we obtain

\[
\beta^2_G \approx \frac{1224}{E_{21^+} A^{7/3}}.
\] (4)

Hence

\[
\beta = \beta_G \left( \frac{9 - \sqrt{81 - 72 \sin^2(3\gamma)}}{4 \sin^2(3\gamma)} \right)^{1/2}
\] (5)
The interacting boson model

1. Nuclear collective excitations are described in terms of s and d bosons.
2. Spectrum generating algebra for the nucleus is U(6). All physical observables (hamiltonian, transition operators) are expressed in terms of the generators of U(6).
3. Formally, nuclear structure is reduced to solving the problem of N interacting s and d bosons.
There are several equivalent ways of writing Hamiltonian $H$. The most general Hamiltonian that has been used to calculate the level energies is

$$H = \epsilon n_d + a_0 P^\dagger . P + a_1 L . L + a_2 Q . Q + a_3 T_3 . T_3 + a_4 T_4 . T_4$$

(6)

where

$$n_d = (d^\dagger . \tilde{d})$$

$$P = \frac{1}{2} (\tilde{d} . \tilde{d}) - \frac{1}{2} (\tilde{s} . \tilde{s})$$

$$L = \sqrt{10} \left[ d^\dagger \times \tilde{d} \right]$$

$$Q = \left[ d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d} \right] - \frac{1}{2} \sqrt{7} \left[ d^\dagger \times \tilde{d} \right]$$

$$T_3 = \left[ d^\dagger \times \tilde{d} \right]$$

$$T_4 = \left[ d^\dagger \times \tilde{d} \right]$$
Usually the first four terms, namely the boson energy term, and the $a_0$, $a_1$, and $a_2$ terms are adequate for the phenomenological fit to the low energy spectrum of the nucleus. The computer program code PHINT was used for the construction of the IBM Hamiltonian. The input parameters EPS, ELL, QQ, OCT and HEX are related to the coefficients $\epsilon$, $a_0$, $a_1$, $a_2$, $a_3$, $a_4$ respectively, where $EPS = \epsilon, PAIR = a_0/2, ELL = 2a_1, QQ = 2a_2, OCT = a_3/5, HEX = a_4/5$

Interacting boson model has a very definite group structure, that of the group U(6), different reductions of U(6) gives three dynamical symmetry limits known as harmonic oscillator, deformed rotor and asymmetric deformed rotor which are labeled by U(5), SU(3) and O(6) respectively.
\[ U(6) \supset U(5) \supset O(5) \supset O(3) \]
\[ U(6) \supset SU(3) \supset O(3) \]
\[ U(6) \supset O(6) \supset O(5) \supset O(3) \]

The energy eigenvalue for three chains are as

\[ E^{(I)}(N, n_d, \nu, n_\Delta, L) = \epsilon n_d + \alpha \frac{1}{2} n_d(n_d - 1) + \beta [n_d(n_d + 3) - \nu(\nu + 3)] + \gamma [L(L + 1) - 6n_d] \]

\[ E^{(II)}(N, \lambda, \mu, L) = \left( \frac{3}{4} \kappa - \kappa' \right) L(L + 1) - \kappa [\lambda^2 + \mu^2 + \lambda \mu + 3(\lambda + \mu)] \]

\[ E^{(III)}(N, \sigma, \tau, \nu_\Delta, L) = A \frac{1}{4} (N - \sigma)(N + \sigma + 4) + B \frac{1}{6} \tau(\tau + 3) + CL(L + 1) \]
There is a long-standing debate about the nature of the spectra characterizing Os isotopes. Some group consider these nuclei as being $\gamma$-soft, while others as asymmetric rotor which assumes rigidity in the $\gamma$ degrees freedom. The predicted value of $\gamma$ for $^{188,190,192}$Os are $20^\circ$, $22^\circ$, $25^\circ$, respectively, which might be considered closer to $\gamma=30^\circ$ picture of the ideal triaxial liquid drop.

The predicted value of the $\gamma$ for $^{228,230}$Th are $10^\circ$, $11^\circ$ respectively. The reason to mention these nuclei is that these nuclei satisfy the well known most distinctive signature of the triaxial rigid rotor equation relating to the energies of three particular states.
Casten’s symmetry triangle in its geometric representation describe the all three limits.
Excitation energies for ground, $\beta$ and $\gamma$ bands of $^{190}\text{Os}$, calculated by IBM and compared with CSM and corresponding experimental data.
Figure 2
Excitation energies for ground, $\beta$ and $\gamma$ bands of $^{192}\text{Os}$, calculated by IBM and compared with CSM and corresponding experimental data.
Table 1
Some $\text{B(E2)}$ values for $^{188-190}\text{Os}$ obtained with IBM compared with the corresponding Experimental data and CSM approach.

<table>
<thead>
<tr>
<th>BE(2)</th>
<th>Exp</th>
<th>IBM</th>
<th>CSM</th>
<th>Exp</th>
<th>IBM</th>
<th>CSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2_g \rightarrow 0_g$</td>
<td>0.502</td>
<td>0.504</td>
<td>0.456</td>
<td>0.468</td>
<td>0.454</td>
<td>0.360</td>
</tr>
<tr>
<td>$4_g \rightarrow 2_g$</td>
<td>0.776</td>
<td>0.713</td>
<td>0.744</td>
<td>0.623</td>
<td>0.654</td>
<td>0.579</td>
</tr>
<tr>
<td>$6_g \rightarrow 4_g$</td>
<td>0.843</td>
<td>0.757</td>
<td>0.918</td>
<td>0.679</td>
<td>0.702</td>
<td>0.708</td>
</tr>
<tr>
<td>$8_g \rightarrow 6_g$</td>
<td>0.927</td>
<td>0.734</td>
<td>1.062</td>
<td>0.814</td>
<td>0.686</td>
<td>0.814</td>
</tr>
<tr>
<td>$2\gamma \rightarrow 0_g$</td>
<td>0.047</td>
<td>0.0078</td>
<td>0.165</td>
<td>0.039</td>
<td>0.0040</td>
<td>0.202</td>
</tr>
<tr>
<td>$2\gamma \rightarrow 2_g$</td>
<td>0.150</td>
<td>0.150</td>
<td>0.227</td>
<td>0.289</td>
<td>0.155</td>
<td></td>
</tr>
<tr>
<td>$4\gamma \rightarrow 2_g$</td>
<td>0.009</td>
<td>0.0028</td>
<td>0.163</td>
<td>0.005</td>
<td>0.003</td>
<td>0.220</td>
</tr>
<tr>
<td>$4\gamma \rightarrow 4_g$</td>
<td>0.036</td>
<td>0.0032</td>
<td>0.001</td>
<td>0.229</td>
<td>0.245</td>
<td>0.229</td>
</tr>
<tr>
<td>$6\gamma \rightarrow 4_g$</td>
<td>0.001</td>
<td>0.001</td>
<td>0.194</td>
<td>0.003</td>
<td>0.009</td>
<td>0.269</td>
</tr>
<tr>
<td>$6\gamma \rightarrow 6_g$</td>
<td>0.164</td>
<td>0.0029</td>
<td>0.227</td>
<td>0.238</td>
<td>0.259</td>
<td>0.270</td>
</tr>
</tbody>
</table>
Figure 3
(a) Staggering $S(J)$ for the IBM and compared with vibrator and $\gamma$-soft, the E(5) critical point symmetry, and the E(5)-$\beta^{2n}$ ($n=2,4$) models. (b) same for the Davydov model, the Z(5) and Z(4) models as well as for their analogs Z(5)-$\beta^2$ and Z(4)-$\beta^2$. (c) Same for the axially symmetric limit, the X(5) critical point symmetry, the X(5)-$\beta^{2n}$ ($n=1,2$) models and the exactly separable models ES-X(5)-$\beta^2$. 
Figure 4
(a) Staggering $S(J)$ for the IBM and compared with vibrator and $\gamma$-soft, the E(5) critical point symmetry, and the E(5)-$\beta^{2n}$ ($n=2,4$) models. (b) same for the Davydov model, the Z(5) and Z(4) models as well as for their analogs Z(5)-$\beta^{2}$ and Z(4)-$\beta^{2}$. (c) Same for the axially symmetric limit, the X(5) critical point symmetry, the X(5)-$\beta^{2n}$ ($n=1,2$) models and the exactly separable models ES-X(5)-$\beta^{2}$.
Figure 5
(a) Staggering S(4) for the IBM and compared with vibrator and $\gamma$-soft, Davydov and other geometrical models.
Figure 6
Zig-Zag phase transition in S(J)-J plot and $2J/h^2-(h\omega^2)$ plot for $^{188-192}$Os and $^{228-230}$Th nuclei.
Figure 7 Back bending in $^{188-192}$Os and $^{228-230}$Th isotopes.
The experimental and calculated difference $\Delta E_1$ and $\Delta E_2$ where $\Delta E_1 = E(3^+_1) - [E(2^+_1) + E(2^+_2)]$ for triaxial nucleus and $\Delta E_2 = E(3^+_1) - [2E(2^+_1) + E(4^+_1)]$ for $\gamma$-soft nucleus.

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>$^{188}$Os</th>
<th>$^{190}$Os</th>
<th>$^{192}$Os</th>
<th>$^{228}$Th</th>
<th>$^{230}$Th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp.$\Delta E_1$</td>
<td>1.90</td>
<td>11.3</td>
<td>4.47</td>
<td>4.26</td>
<td>46.5</td>
</tr>
<tr>
<td>Exp.$\Delta E_2$</td>
<td>1.96</td>
<td>165.2</td>
<td>301.4</td>
<td>719.8</td>
<td>600.5</td>
</tr>
<tr>
<td>Th.$\Delta E_1$</td>
<td>4.5</td>
<td>13.1</td>
<td>6.3</td>
<td>12.9</td>
<td>50.9</td>
</tr>
<tr>
<td>Th.$\Delta E_2$</td>
<td>60.5</td>
<td>49.7</td>
<td>260.3</td>
<td>744.5</td>
<td>608.5</td>
</tr>
</tbody>
</table>
Summary

1. The formalism was applied to five nuclei $^{188,190,192}$Os, $^{228}$Th and $^{230}$Th. The comparison with experimental data shows a good agreement between the corresponding results and data. We also compare IBM results with those obtained with SRF and CSM.

2. The experimental energy staggering in $\gamma$ bands of $^{188,190,192}$Os, $^{228}$Th and $^{230}$Th isotope is investigated as a signature for the $\gamma$ dependence of the geometric potential. We note that oscillation amplitude of $S(J)$ is increasing with $J$. The staggering quantity, $S(4)$, is also investigated as a function of the collectivity using $R_{4/2}$ ratio.
THANKS FOR YOUR ATTENTION